Honors Algebra II — Name KEY

LT 8-1: Characteristics of Polynomials Day 1 — Period____

Using the characteristics that we discussed in class today, fill in the blanks.

1) An __Odd__ degree polynomial must have at least one real zero.

2) A polynomial function is written in __Standard__ __form__ if its terms are written in descending order of exponents from left to right.

3) The __Leading__ __Coefficent__ is the number in front of the term with the highest exponent in the polynomial.

4) A __monomial__ is a polynomial with one term, a __binomial__ has two terms, and a __trinomial__ has three terms.

5) It is possible for an __even__ degree polynomial to have no real zeros.

6) The __leading__ __term__ __test__ is used to determine the end behavior of the graph of a polynomial function.

Write each polynomial in standard form and state the degree, type, leading coefficient, and the end behavior. The first example has been done for you.

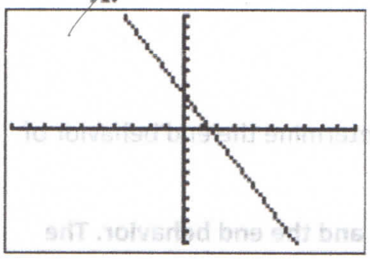
	Standard Form	Degree	Classify by Degree	Classify by Number of Terms	LC	End Behavior
Example: $y = 7 - 3x$	$y = -3x + 7$	1	Linear	Binomial	-3	As $x \to -\infty, y \to \infty$ As $x \to \infty, y \to -\infty$
7) $f(x) = 2x - x^3 + 8$	$f(x) = -x^3 + 2x + 8$	3	Cubic	Trinomial	-1	As $x \to -\infty$, $f(x) \to \infty$ As $x \to \infty$, $f(x) \to -\infty$
8) $y = 3x^2 + x^3 - (x^3 + x^2)$ $3x^2 + \cancel{x^3} - \cancel{x^3} - x^2$	$y = 2x^2$	2	Quadratic	Monomial	2	As $x \to -\infty$, $y \to \infty$ As $x \to \infty$, $y \to \infty$
9) $y = (2x)^3 + 3x - 1$	$y = 8x^3 + 3x - 1$	3	Cubic	Trinomial	8	As $x \to -\infty$, $y \to -\infty$ As $x \to \infty$, $y \to \infty$
10) $f(x) = (x + 2)^2 + 3$ $= x^2 + 4x + 4 + 3$	$f(x) = x^2 + 4x + 7$	2	Quadratic	Trinomial	1	As $x \to -\infty$, $f(x) \to \infty$ As $x \to \infty$, $f(x) \to \infty$
11) $y = (2 + x)(2 - x) - 4$ $= 4 - x^2 - 4$	$y = -x^2$	2	Quadratic	Monomial	-1	As $x \to -\infty$, $y \to -\infty$ As $x \to \infty$, $y \to -\infty$
12) $f(x) = 3(x + 1)^2 - 3x^2$ $= 3(x^2 + 2x + 1) - 3x^2$	$f(x) = 6x + 3$	1	Linear	Binomial	6	As $x \to -\infty$, $f(x) \to -\infty$ As $x \to \infty$, $f(x) \to \infty$
13) $g(x) = 2x - 2(x - 3)$ $= 2x - 2x + 6$	$g(x) = 6$	0	Constant	Monomial	6	As $x \to -\infty$, $g(x) \to 6$ As $x \to \infty$, $g(x) \to 6$

Describe the end behavior of the graph of the polynomial function without graphing.

14) $y = 4x - 2 \boxed{+ 5x^5}$ LT As $x \to -\infty, y \to$ $-\infty$ and as $x \to \infty, y \to$ ∞	15) $y = \boxed{-5x^3} + 2x$ LT As $x \to -\infty, y \to$ ∞ and as $x \to \infty, y \to$ $-\infty$	LT 16) $y = -2x \boxed{- 12x^6} + 5$ As $x \to -\infty, y \to$ $-\infty$ and as $x \to \infty, y \to$ $-\infty$
17) $y = 6 - 2x + 6x^2 \boxed{- 12x^9}$ LT As $x \to -\infty, y \to$ ∞ and as $x \to \infty, y \to$ $-\infty$	18) $y = \cancel{1} - x^6 \cancel{- 1} + 2x^6 = \boxed{x^6}$ LT As $x \to -\infty, y \to$ ∞ and as $x \to \infty, y \to$ ∞	19) $y = x(x - 2)(x + 3)(-2x - 5)$ $LT = -2x^4$ As $x \to -\infty, y \to$ $-\infty$ and as $x \to \infty, y \to$ $-\infty$

Match the polynomial function with its graph without using a graphing calculator.

A.



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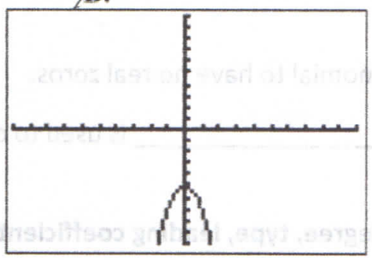

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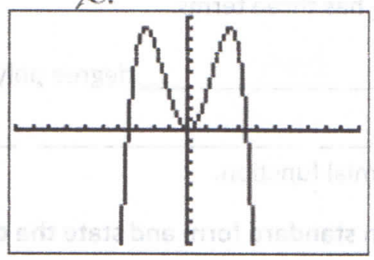

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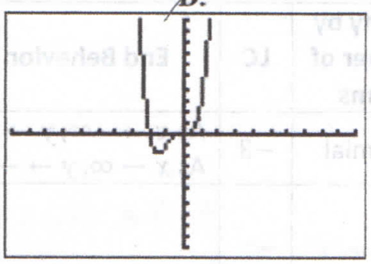

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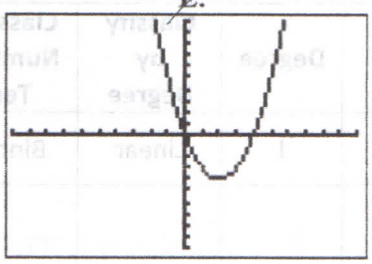

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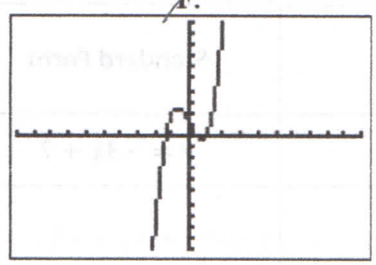

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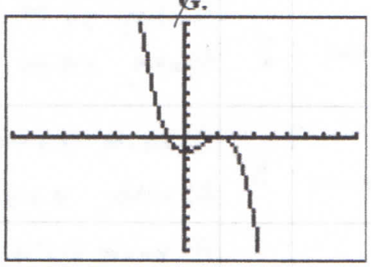

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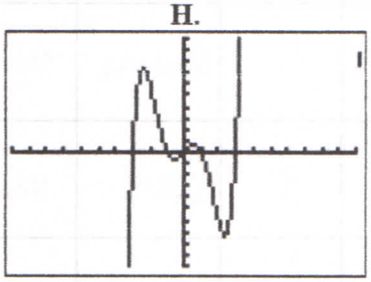
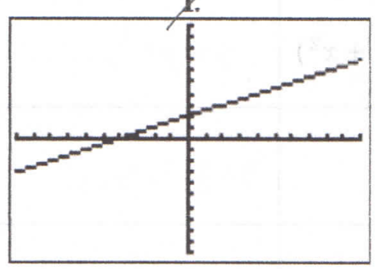


I.



E 20) $y = x^2 - 4x$

F 21) $y = 2x^3 - 3x + 1$

G 22) $y = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$

I 23) $y = \frac{1}{2}x + 2$

B 24) $y = -2x^2 - 5$

C 25) $y = 3x^2 - \frac{1}{4}x^4$

A 26) $y = 3 - 2x$

D 27) $y = 2x^3 + x^4$

H 28) $y = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$

Name KEY

LT 8-1: Characteristics of Polynomials Day 2

Period____

State the information for the given polynomials. Then, provide a sketch of the function.

1) $P_1(x) = (x-2)(x+5)^2$ LT $= x^3$

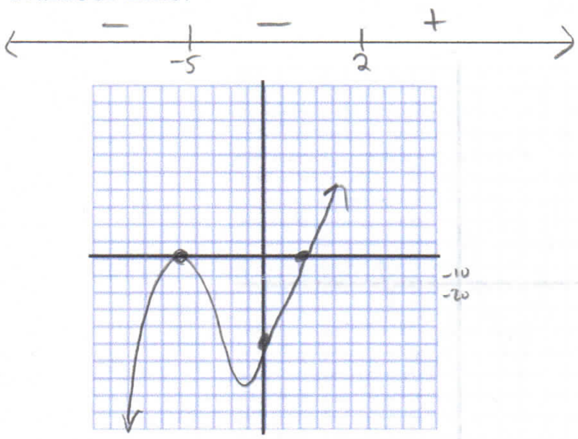
y-intercept: $(0, -50)$

x-intercept(s): $(-5, 0), (2, 0)$

Degree: 3 Bounce Points: $(-5, 0)$

End Behavior/Orientation: As $x \to -\infty$, $P_1(x) \to -\infty$; As $x \to \infty$, $P_1(x) \to \infty$

Number Line:



2) $P_2(x) = 2(x-2)(x+2)(x-3)$ LT $= 2x^3$

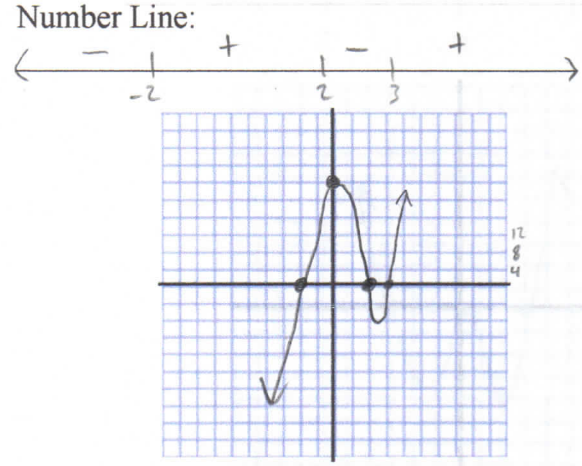
y-intercept: $(0, 24)$

x-intercept(s): $(-2, 0), (2, 0), (3, 0)$

Degree: 3 Bounce Points: None

End Behavior/Orientation: As $x \to -\infty$, $P_2(x) \to -\infty$; As $x \to \infty$, $P_2(x) \to \infty$

Number Line:



3) $P_3(x) = -2(x+3)^2(x+1)(x-1)(x-5)$ LT $= -2x^5$

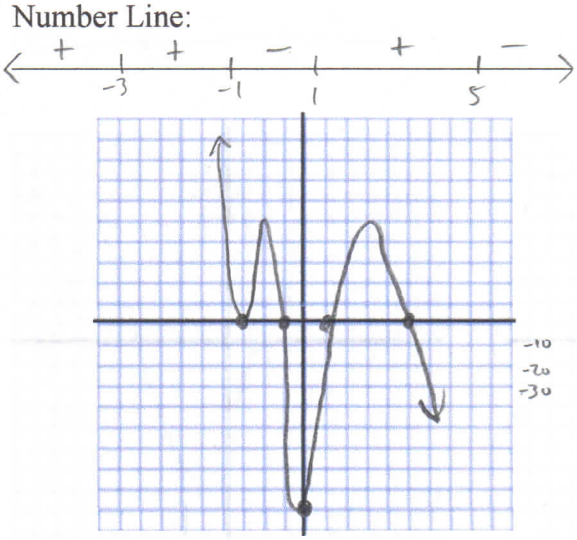
y-intercept: $(0, -90)$

x-intercept(s): $(-3, 0)\ (-1, 0)\ (1, 0)\ (5, 0)$

Degree: 5 Bounce Points: $(-3, 0)$

End Behavior/Orientation: As $x \to -\infty$, $P_3(x) \to \infty$; As $x \to \infty$, $P_3(x) \to -\infty$

Number Line:



4) $P_4(x) = -0.1x(x+4)^3$ LT $= -0.1x^4$

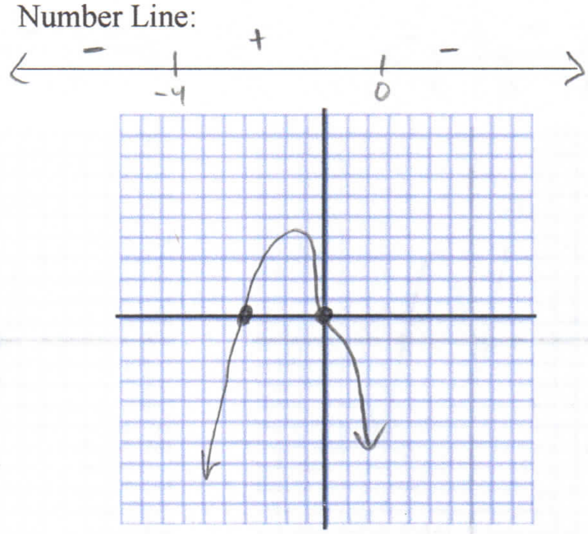
y-intercept: $(0, 0)$

x-intercept(s): $(-4, 0)\ (0, 0)$

Degree: 4 Bounce Points: None

End Behavior/Orientation: As $x \to -\infty$, $P_4(x) \to -\infty$; As $x \to \infty$, $P_4(x) \to -\infty$

Number Line:



5) $P_5(x) = x^4 - 9x^2 = x^2(x^2-9) = x^2(x-3)(x+3)$

y-intercept: $(0,0)$ $LT = x^4$

x-intercept(s): $(-3,0)$ $(0,0)$ $(3,0)$

Degree: 4

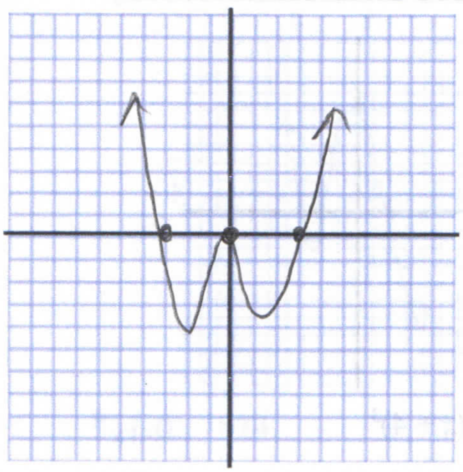
Bounce Points: $(0,0)$

End Behavior/Orientation: As $x \to -\infty$, $P_5(x) \to \infty$; As $x \to \infty$, $P_5(x) \to \infty$

Number Line:

$+ \quad - \quad - \quad +$

$-3 \quad 0 \quad 3$



6) $P_6(x) = 0.2x(x + 1)(x - 3)(x + 4)$ $LT = 0.2x^4$

y-intercept: $(0,0)$

x-intercept(s): $(-4,0)$ $(-1,0)$ $(0,0)$ $(3,0)$

Degree: 4

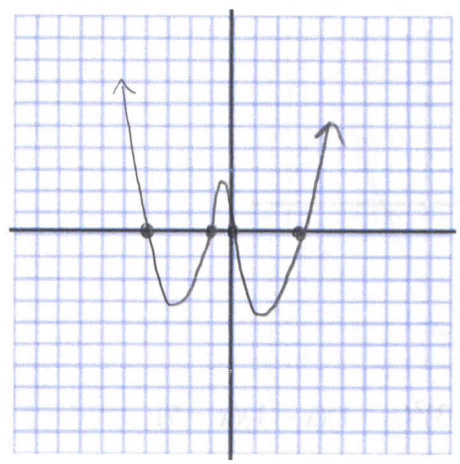
Bounce Points: None

End Behavior/Orientation: As $x \to -\infty$, $P_6(x) \to \infty$; As $x \to \infty$, $P_6(x) \to \infty$

Number Line:

$+ \quad - \quad + \quad - \quad +$

$-4 \quad -1 \quad 0 \quad 3$



7) Without using a calculator, sketch rough graphs of the following functions.

a) $P(x) = -x(x + 1)(x - 3)$

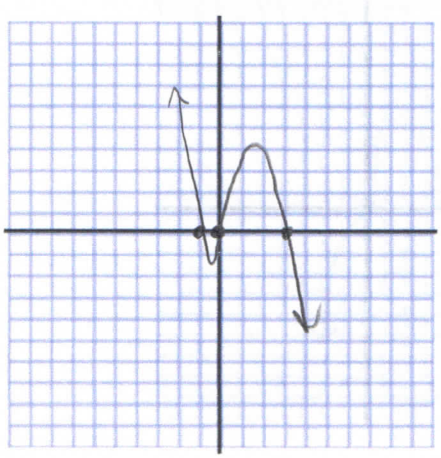
Zeros: $-1, 0, 3$

LT: $-x^3$

No bounce points

$+ \quad - \quad + \quad -$

$-1 \quad 0 \quad 3$



b) $P(x) = (x - 1)^2(x + 2)(x - 4)$

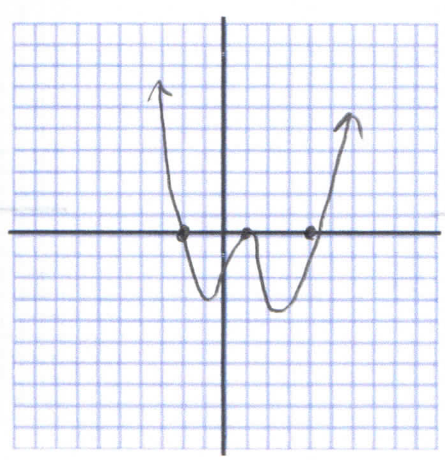
Zeros: -2, 1 mult. 2, 4

LT: x^4

Bounce point: $(1,0)$

$+ \quad - \quad - \quad +$

$-2 \quad 1 \quad 4$



c) $P(x) = (x + 2)^3(x - 4)$

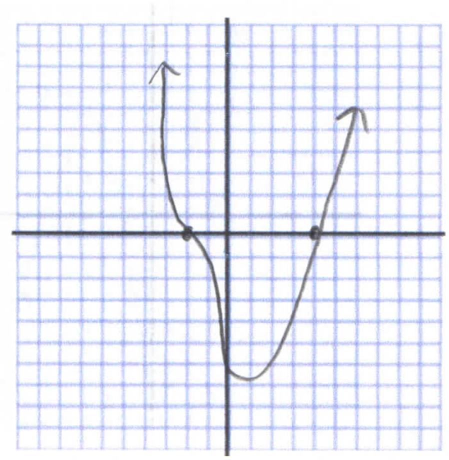
Zeros: -2 mult. 3, 4

LT: x^4

No bounce points

$+ \quad - \quad +$

$-2 \quad 4$



Honors Algebra II Name KEY

LT 8-1: Characteristics of Polynomials Day 3 Period____

Write the following polynomials in standard form:

1) $f(x) = x(x+3)^2$

$= x(x+3)(x+3)$

$= x(x^2+6x+9)$

$\boxed{f(x) = x^3+6x^2+9x}$

2) $g(x) = (x+1)(x+2)(x+3)$

$= (x^2+3x+2)(x+3)$

$= x^3+3x^2+2x+3x^2+9x+6$

$\boxed{g(x) = x^3+6x^2+11x+6}$

Write a polynomial function in standard form with the given zeros.

3) $x = -2, 0, 1$

$f(x) = (x+2)(x-1)(x)$

$= x(x^2+x-2)$

$\boxed{f(x) = x^3+x^2-2x}$

4) $x = 3$ multiplicity 2

$f(x) = (x-3)^2$

$= (x-3)(x-3)$

$\boxed{f(x) = x^2-6x+9}$

5) $x = -2, 0$ multiplicity 3, 2

$f(x) = x^3(x+2)(x-2)$

$= x^3(x^2-4)$

$\boxed{f(x) = x^5-4x^3}$

6) $x = -4, -3, 0, 3, 4$

$f(x) = x(x+4)(x-4)(x+3)(x-3)$

$= x(x^2-16)(x^2-9)$

$= x(x^4-25x^2+144)$

$\boxed{f(x) = x^5-25x^3+144x}$

7) Write a polynomial equation for a graph that passes through the point $(-1, 60)$ and has three x-intercepts: $(-4, 0)$, $(1, 0)$, and $(3, 0)$.

$f(x) = a(x+4)(x-1)(x-3)$

$= a(x^2+3x-4)(x-3)$

$= a(x^3+3x^2-4x-3x^2-9x+12)$

$f(x) = a(x^3-13x+12)$

$f(-1) = a((-1)^3-13(-1)+12) = 60$

$a(24) = 60$

$a = \frac{60}{24} = \frac{5}{2}$

$f(x) = \frac{5}{2}(x^3-13x+12)$

$\boxed{f(x) = \frac{5}{2}x^2 - \frac{65}{2}x + 30}$

8) Write a polynomial equation for a graph that has three x-intercepts: $(-5, 0)$, $(3, 0)$ and $(1, 0)$ and it passes through the point $(4, 108)$.

$f(x) = a(x+5)(x-3)(x-1)$

$= a(x^2+2x-15)(x-1)$

$= a(x^3+2x^2-15x-x^2-2x+15)$

$f(x) = a(x^3+x^2-17x+15)$

$f(4) = a(4^3+4^2-17(4)+15) = 108$

$a(80-68+15) = 108$

$27a = 108$

$a = 4$

$f(x) = 4(x^3+x^2-17x+15)$

$\boxed{f(x) = 4x^3+4x^2-68x+60}$

9) Write a polynomial equation for a graph that has x-intercepts at $(-2, 0)$ and $(3, 0)$, a bounce point at $(-4, 0)$ and passes through the point $(5, 25)$.

$f(x) = a(x+2)(x-3)(x+4)^2$

$= a(x+2)(x-3)(x+4)(x+4)$

$= a(x^2-x-6)(x^2+8x+16)$

$= a(x^4-x^3-6x^2+8x^3-8x^2-48x+16x^2-16x-96)$

$f(x) = a(x^4+7x^3+2x^2-64x-96)$

$f(5) = a(5^4+7(5)^3+2(5)^2-64(5)-96) = 25$

$a(625+875+50-320-96) = 25$

$a(1550-416) = 25$

$a(1134) = 25$

$a = \frac{25}{1134}$

$\boxed{f(x) = \frac{25}{1134}(x^4+7x^3+2x^2-64x-96)}$

Does not have to be in Standard form

10) A rectangular box is $2x + 3$ units long, $2x - 3$ units wide, and $3x$ units high. Express its volume as a polynomial in standard form.

$V = LWH$

$V(x) = (2x+3)(2x-3)(3x)$

$= (4x^2+6x-6x-9)(3x)$

$= (4x^2-9)(3x)$

$\boxed{V(x) = 12x^3-27x}$