Warm-Up | The Quadratic Formula

Lesson Goals

By the end of this lesson, you should be able to:

- Find real and ________ solutions of quadratic equations using the quadratic formula.
- Use the discriminant to determine the ________ and ________ of roots of a quadratic equation.

Words to Know

Fill in this table as you work through the lesson. You may also use the glossary to help you.

discriminant	the ________ found in the quadratic formula, used to determine the number and type of solutions to a quadratic equation
quadratic formula	a ________ for finding the solutions of a ________ equation in standard form

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The Quadratic Formula

The quadratic formula is really useful when you need to solve a quadratic equation that you can't solve using the other methods that you already have.

The **quadratic formula** is a formula for finding the solutions of a quadratic equation in the form ______ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

How to Use the Quadratic Formula

To solve a quadratic equation:

Step 1 Write equation in standard form: $0 = ax^2 + bx + c$

Step 2 Identify the values of ___, ___, and ___

Step 3 ______ the values of a, b, and c into the quadratic formula

Step 4 Simplify the expression

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Example: Approximate the zeroes of $y = -16x^2 + 32x - 10$. Round to the nearest hundredth. It's already in standard form.

Identify a, b, and c.

$$a = -16, b = \boxed{}, c = \boxed{}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(32) \pm \sqrt{(32)^2 - 4(-16)(-10)}}{2(-16)}$$

$$x = \frac{-32 \pm \sqrt{1024 - 640}}{-32}$$

$$x = \frac{-32 \pm \sqrt{384}}{-32}$$

$$x = \frac{-32 \pm 19.59}{-32}$$

$$x = \frac{-32 + 19.59}{-32}$$

$$x \approx \frac{-12}{-32}$$

$$x \approx \boxed{}$$

$$x = \frac{-32 - 19.59}{-32}$$

$$x \approx \frac{-52}{-32}$$

$$x \approx \boxed{}$$

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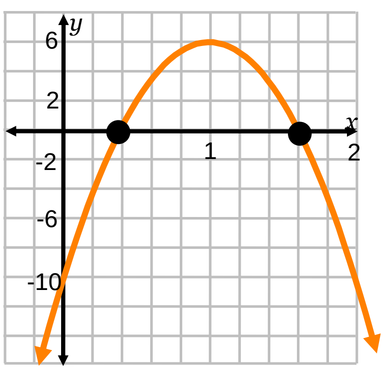
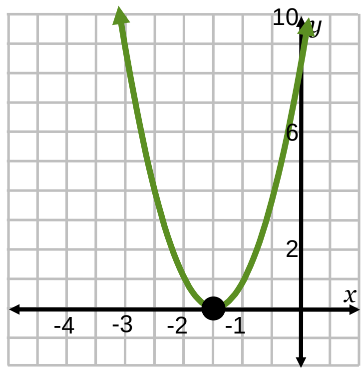
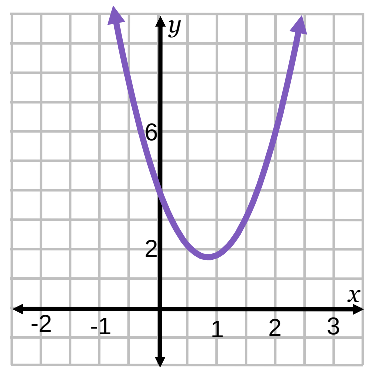
Graphical Representations of Solution Possibilities







real solutions

$x \approx 0.38$ and 1.61

The graph crosses the x-axis at two points

One real solution

$x = -1.5$

The graph touches the x-axis at one point

real solution

$x = ??$

The parabola never touches the x-axis

The solutions are therefore

.

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Quadratic Equations with No Real Solution

Example: Solve $-3x^2 - x = -6x + 4$.

Get the equation in the standard form.

$$0 = ax^2 + bx + c$$

$$0 = 3x^2 - 6x + x + 4$$

$$0 = \boxed{}$$

Identify a, b, and c. $a = \boxed{}$, $b = -5$, $c = 4$

Use the quadratic formula to solve.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{25 - 48}}{6}$$

$$x = \frac{5 \pm \sqrt{-23}}{6}$$

$$x = \frac{5 \pm \sqrt{23}i}{6}$$

This quadratic function has two roots, and they're $\boxed{}$.

The two roots are $x = \frac{5 + \sqrt{23}i}{6}$ and $x = \frac{5 - \sqrt{23}i}{6}$.

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The Discriminant

The **discriminant** is the radicand found in the quadratic formula.

- $b^2 - 4ac \; \boxed{} \; 0$

  2 real solutions

- $b^2 - 4ac \; \boxed{} \; 0$

  1 real solution

- $b^2 - 4ac \; \boxed{} \; 0$

  0 real solutions

Quadratic formula

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

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How to Use the Discriminant

Example: Describe the zeroes of the function $y + 7x - 1 = 9x^2 + 3x + 5$.

Get the equation in the standard form.

$$y = ax^2 + bx + c$$

$$y = 9x^2 - 4x + 6$$

Identify a, b, and c. $a = \boxed{}$, $b = -4$, $c = 6$

Since we're not solving, we don't need the quadratic formula. All we need is:

$$b^2 - 4ac$$

$$(-4)^2 - 4(9)(6)$$

$$16 - 216$$

$$-200$$

- Since we have a $\boxed{}$ under the radical, we have an imaginary number.
- Our roots are going to be complex.
- There are $\boxed{}$ real roots to this particular problem.

Summary | The Quadratic Formula

Lesson Question

How can a formula be used to solve a quadratic equation or to predict the nature of the solutions?

Answer:

Review: Key Concepts

The **quadratic formula** is

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

- $\boxed{}$ of

  $0 = ax^2 + bx + c$

- x-intercepts of $y = ax^2 + bx + c$

The **discriminant** is

$\boldsymbol{b^2 - 4ac}$

- $b^2 - 4ac > 0$

  $\boxed{}$ real solutions

- $b^2 - 4ac = 0$

  1 real solution

- $b^2 - 4ac < 0$

  $\boxed{}$ real solutions

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Review: Common Problem Types

To solve a quadratic equation using the quadratic formula:

1. Write the equation in ____________ form: $0 = ax^2 + bx + c$.
2. Identify the values of a, b, and c from the equation.
3. Substitute the values of a, b, and c into the quadratic ____________.
4. Simplify the expression for x.

To determine the nature of a quadratic's solutions:

1. Follow the same steps as above, but use the discriminant only.
2. Assess the sign to identify the ____________ and ____________ of solutions.

Summary | The Quadratic Formula

Use this space to write any questions or thoughts about this lesson.